COURSE DESCRIPTION CARD - SYLLABUS

Course name
Linear algebra with analitic geometry I [S1MNT1>ALzGA1]

Course

Field of study
Mathematics of Modern Technologies

Year/Semester
1/1

Area of study (specialization)
–

Profile of study
general academic

Level of study
first-cycle

Course offered in
Polish

Form of study
full-time

Requirements
compulsory

Number of hours

Lecture
30

Laboratory classes
0

Other
0

Tutorials
30

Projects/seminars
0

Number of credit points

5,00

Coordinators

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Lecturers

Prerequisites

Basic knowledge of the high school. Ability to efficiently perform algebraic operations, knowledge of number sets and properties of operations. He is aware of the need to expand his competences and is ready to cooperate

Course objective

Learning the basics of the calculus of complex numbers. Getting to know the matrix calculus and applying it to solve systems of linear equations. Getting to know the basics of the theory of linear spaces and linear operators, acquiring the ability to solve the eigenvalue problem of linear operator. Using the calculus of vector algebra to analyze a line and a plane in the space

Course-related learning outcomes

Knowledge:
• has knowledge about the notion of a complex number in different forms, about the basic concepts of the matrix calculus, the theory of linear spaces and linear operators, understands the proofs of more important selected theorems or the ideas of proofs from the above area [K_W01(P6S_WG), K_W03(P6S_WG)];

• has knowledge of the basic concepts of vector algebra, is able to recognize the equations of a line and a plane in the space [K_W01(P6S_WG), K_W03(P6S_WG)].

Skills:
• has the ability to calculate determinants, is able to determine the rank of the matrix, inverse matrix, use matrix calculus to solve systems of linear equations, recognize linear subspaces and the dimension of a linear space, solve the eigenvalue problem of a linear operator given by a matrix [K_U01(P6S_UW)];
• can determine the equation of a line and a plane in space with the use of vector algebra, use the basic calculus of complex numbers [K_U01(P6S_UW)].

Social competences:
• canthinkandbehaveingoodmathematicalmannerintheareaoflinearalgebraandanaliticalgeometry [K_K01(P6S_KK)];
• knows the limitation of own knowledge and understand the need of more far education and the necessity of systematic work [K_K02(P6S_KK)].

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lectures:
Passing threshold: at least 50% of points. The issues for the exam, on the basis of which the questions are prepared, will be sent to students with the use of university electronic systems.
• assessment of knowledge and skills on a written exam that checks the knowledge of notions and the ability to prove theorems and illustrate theories with examples (also possible short practical tasks);
• obtaining additional points for activity during classes, including for the preparing presentations (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or a reference to the location in the history of mathematics) and for comments on improving teaching materials.

Tutorials:
The knowledge acquired during the exercises is verified by two tests carried out on approx. 7 and 15 weeks (alternatively 1 test at the end of the semester). Passing threshold: at least 50% of points.
The rules for completing the course and the exact thresholds for passing the course will be provided to students at the beginning of the semester with the use of university electronic systems.
• continuous assessment - rewarding activity (additional points) manifested in the discussion and co operation in solving practical tasks;
• continuous assessment - rewarding the increase in the ability to use the techniques learned;
• obtaining additional points for activity during classes, including for the preparing presentations (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or a reference to the location in the history of mathematics) and for comments on improving teaching materials;
• active participation in consultations deepening knowledge and directing further work.

Programme content

Update: 01.06.2025r.
Lectures: theoretical issues (definitions, lemmas, theorems, corolaries, algorithms) and suitable examples for the following issues:
• complex numbers (algebraic, trigonometric and exponential forms, operations on complex numbers, algebraic equations);
• number fields, abstract fields. Linear spaces, basis, dimension. Linear transformations (operators), eigenvalues and eigenvectors of a linear transformation;
• matrices, determinants, systems of linear equations, matrix equations, matrix rank, inverse matrix;
• vector algebra (scalar, vector and mixed product of vectors), line and plane in space.
Tutorials: solving practical problems illustrating the concepts discussed and example problems with the use of theoretical machinery from the lecture, e.g.: using the algebraic, trigonometric or exponential form to solve algebraic equations, determining sets on the complex plane, determining the dimension of a linear space, determining the coordinates of an element after changing the basis, studying linear subspaces, studying linearity of the operator and determining the operator's matrix in a fixed basis, solving the eigenproblem of operator, solving matrix equations, calculating determinants, solving systems of linear equations using the Gaussian method, determining the inverse matrix, rank of the matrix, using the calculus of vector algebra in geometry to determine and analyze the equation of a line and a plane.

Course topics

Lecture

Complex numbers

1. algebraic form of a complex number.
2. the trigonometric form of a complex number, derivation from the algebraic form.
3. exponential of a complex number.
4. operations on complex numbers: addition, multiplication and division in algebraic and trigonometric form.
5. verify that (C,+,) is a field.
6. Conjugate complex numbers and properties.
7. Modulus properties of product, quotient of complex numbers.
8. Properties of the argument of the product, quotient and power of complex numbers.

9 Moivre's formula.

10. Trigonometric ringing of complex numbers - theorem with proof and geometric interpretation.
11. algebraic equations in the complex domain and Gauss' theorem.

Matrix calculus part 1

12 Definition of a matrix, types of matrices, examples.

13. operations on matrices: addition of a matrix, multiplication of a matrix by a number, multiplication of a matrix by a matrix (definitions and properties).
14. definition of a system of linear equations.
15. the Gauss elimination method.
16. definition of the determinant of a matrix + application - calculation of the determinant of a matrix of degree 1,2 and 3 .
17. the Laplace expansion theorem for the determinant.

18 Properties of determinants.

19 Cramer's system - definition and theorem .

19a. Inverse matrix- definition and properties.

20 The theorem on the determinant of an inverse matrix.

21. two theorems on the determinant of an inverse matrix.
22. Interpretation of a system of equations (solution) in terms of a matrix (inverse matrix).

Fields and linear spaces

23. definition of abstract field, examples of fields.
24. definition of linear space, examples.
25. linear combination.
26. linear subspace- definition, examples.
27. definition of linear span, examples.
28. Linearly dependent vectors and linearly independent vectors-definition and equivalent conditions,; examples.
29. dimension of a linear space-definition, examples.
30. basis of a linear space-definition, examples.
31. Theorem-Equivalence condition for a system $B=\{e_1, e_2, \ldots e_{n}\}$ to be a basis.

32 The theorem on the complement of a linearly independent system to a basis.

Matrix calculus part 2

33. The space of rows of a matrix- definition and theorem of row-equivalent matrices.
34. rank of matrices- definition and equivalent formulations.
35. Kronecker-Capelli theorem with proof and conclusion on number of solutions.
36. homogeneous systems- definition and discussion of the number of solutions based on Kronecker-Capelli Thm.

Linear operators, values and eigenvectors of a linear operators

37 The definition of a linear operator, examples.

38. Definition of kernel and image of a linear operators.
39. Theorem on the representation of a linear operator by a matrix .
40. definition of eigenvalue and eigenvector.
41. definition of eigenspace and spectrum.
42. Characteristic equation and characteristic polynomial of a matrix- definition.
43. the Cayley-Hamilton theorem.
44. algebraic sum of linear spaces.
45. the codimension of a linear subspace.

46. direct sum of linear spaces. Decomposition theorem.
47. a) The projection operator.
(b) Operator matrix theorem with respect to a new basis (transition matrix from one basis to another). Theorem on the operator matrix with respect to a basis composed of its eigenvectors.
(b) The orthogonal square matrix. Theorem on the orthogonal diagonalizability of a real symmetric square matrix.

Analytical geometry
48 Vector -definition.
49. vector lengths, directional cosines.
50. Rectangular Cartesian coordinate system in space.
51. parallel vectors.
52. perpendicular vectors.
53 Addition of vectors.
54. multiplication of a vector by a number.
55. dot product- definition and properties.
56. cross product- definition and properties.
57. mixed product- definition and properties.
58 The parametric equation of a line l and the equation of a line in the form of a double ratio when given a point $P \in l$ and a vector $v \parallel l$.
59. parametric equation of a line l passing through two points $P_1, P_2 \in l$ - .
60. parametric equation of a straight line l when given planes π_1, π_2 such that $l = \pi_1 \cap \pi_2$.
61. Equation of the plane π when given a point $P \in \pi$ and a vector $n \perp \pi$.
62. equation of plane π when given a point $P \in \pi$ and vectors v_1, v_2 such that $v_1 \parallel \pi$, $v_2 \parallel \pi$ and v_1, v_2 are not parallel to each other..
63 The equation of the plane π passing through 3 non-co-linear points.
64. Equation of a plane π containing 2 lines that are parallel and different.
65. equation of a plane π containing a line l_1 and parallel to line l_2 (lines l_1, l_2 are not parallel).
66. the segmental form of the plane and its relation to the general form.
67 The equation of a plane π containing 2 intersecting lines.
68 The angle of two lines.
69 The common point of two lines.
70 The angle of two planes.
71 The angle of a line and a plane.
72. distance of a point from a straight line.
73. distance of two parallel lines.
74. distance of a point from a plane.
75 Distance of two parallel planes.
76. Distance of two oblique lines.

Tutorials
Solving practical tasks using the material presented in the lecture

Teaching methods

Lectures:
• a lecture conducted on the blackboard in an interactive way with the formulation of questions to a group of students, the lecture supplemented by a computer presentation;
• the activity of students is taken into account (preparation of historical talks on mathematicians related to the presented material, papers on the use of algebra in engineering sciences, presenting evidence left to be done on their own) during classes when issuing the final grade;
• initiating discussions during the lecture;
• theory presented in connection with the current knowledge of students from previous lectures.
Tutorials:
• solving example tasks on the blackboard;
• detailed reviewing of the solutions to the tasks by the tutor and discussion of the comments.

Bibliography

Basic:

- A. I. Kostrykin, Wstęp do algebry, cz.1 Podstawy algebry, PWN, Warszawa 2004;
- A. I. Kostrykin, Wstęp do algebry, cz.2 Algebra liniowa, PWN, Warszawa 2004;
- A. I. Kostrykin, Zbiór zadań z algebry, PWN, Warszawa 2005;
- M. Grzesiak, Liczby zespolone i algebra liniowa, Poznań 1999;
- T. Jurlewicz, Z. Skoczylas, Algebra liniowa 1, Wrocław 2003;
- T. Jurlewicz, Z. Skoczylas, Algebra liniowa 2, Wrocław 2005;
- F. Leja, Geometria analityczna, PWN, Warszawa 1961.

Additional:
- H. Arodź, K. Rościszewski, Zbiór zadań z algebry i geometrii analitycznej dla fizyków, PWN, 1990;
- J. Rutkowski, Algebra liniowa w zadaniach, PWN.

Breakdown of average student's workload

	Hours	ECTS
Total workload	125	5,00
Classes requiring direct contact with the teacher	62	2,50
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	63	2,50